



**Achieve ADP Algebra II  
End-of-Course Exam Content Standards  
with Comments & Examples**

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Core Exam and Optional Modules

October 2007

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**Exam Core Content:**

**O: Operations on Numbers and Expressions**

- O1. Real numbers
- O2. Complex numbers
- O3. Algebraic expressions

**E: Equations and Inequalities**

- E1. Linear equations and inequalities
- E2. Nonlinear equations and inequalities

**P: Polynomial and Rational Functions**

- P1. Quadratic functions
- P2. Higher-order polynomial and rational functions

**X: Exponential functions**

- X1. Exponential functions

**F: Function Operations and Inverses**

- F1. Function operations and composition
- F2. Inverse functions
- F3. Piecewise functions

**Optional Module Content:**

**S: Data and Statistics**

- S1. Summarization and comparison of data sets
- S2. Interpretation and communication through data

**PR: Probability**

- PR1. Permutations, combinations, and probability
- PR2. Probability distributions

**L: Logarithmic Functions**

- L1. Logarithmic expressions and equations
- L2. Logarithmic functions

**T: Trigonometric Functions**

- T1. Trigonometric functions

**M: Matrices**

- M1. Matrix arithmetic
- M2. Solving systems of equations using matrices
- M3. Matrix transformations
- M4. Vectors

**C: Conic Sections**

- C1. Conic sections

**I: Sequences and Series**

- I1. Arithmetic and geometric sequences and series
- I2. Other types of iteration and recursion

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**About this document:** There are two versions of the Algebra II End-of-Course Exam Content Standards, one containing the standards, objectives and benchmarks, and this expanded version of the same document, which includes a second column to the right of the benchmarks with explanatory comments and examples. These comments and examples are meant to add clarity to the meaning of the benchmarks for teachers and test item writers. Examples are provided only when necessary for clarity and are not meant to be exhaustive or to be used as sample test items. In some instances, in the standards and in the explanations, the word “including” is used followed by a list. The word “including” does not translate to “all inclusive” but rather means “including but not limited to.”

**Background:** The American Diploma Project (ADP) Network includes 30 states dedicated to making sure every high school graduate is prepared for college and work. Together, Network members are responsible for educating nearly 60 percent of all U.S. public school students. In each state, governors, state superintendents of education, business executives, and college and university leaders are working to restore value to the high school diploma by raising the rigor of high school standards, assessments, and curriculum and better aligning these expectations with the demands of postsecondary education and work.

In May 2005, leaders from several of the ADP Network states began to explore the possibility of working together, with support from Achieve, to develop a common end-of-course exam at the Algebra II level. These states were planning to require or strongly encourage students to take an Algebra II level course in order to better prepare them for college and careers, as Algebra II or its equivalent serves as a gateway course for higher education and teaches quantitative reasoning skills important for the workplace. State leaders recognized that using a common end-of-course test would help ensure a consistent level of content and rigor in classes within and across their respective states. They also understood the value of working collaboratively on a common test: the potential to create a high quality test faster and at lower cost to each state, and to compare their performance and progress with one another.

The ADP Algebra II End-of-Course Exam serves three main purposes:

1. **To improve curriculum and instruction.** The test will help classroom teachers focus on the most important concepts and skills in an Algebra II or equivalent class and identify areas where the curriculum needs to be strengthened. Teachers will get test results back within three weeks of when the test is administered, which will provide sufficient time to make the necessary adjustments for the next year’s course.
2. **To help colleges determine if students are ready to do credit-bearing work.** Because the test is aligned with the ADP mathematics benchmarks, it will measure skills students need to enter and succeed in first-year, credit-bearing math courses. Postsecondary institutions will be able to use the results of the test to tell high school students whether they are ready for college-level work, or if they have content and skill gaps that need to be filled before they enroll in college. This information should help high schools better prepare their students for college, and reduce the need for colleges to provide costly remediation courses.
3. **To compare performance and progress among the participating states.** Having agreed on the core content expectations for courses at the Algebra II level, states are interested in tracking student performance over time. Achieve will issue a report each year comparing performance and progress among the participating states. This report will help state education leaders, educators and the public assess performance, identify areas for improvement, and evaluate the impact of state strategies for improving secondary math achievement.

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The Algebra II End-of-Course Exam will consist of a common core, which will be taken by students across participating states. States not part of the development group may also decide to purchase and administer this test. In addition to the core algebraic content, seven optional modules will be available to states to enrich the core with content which may currently be in their Algebra II or equivalent course that is deemed important to colleges and the workplace. The content in the core and modules is described below.

**The Core Algebra II End-of-Course Exam:** The core Algebra II End-of-Course Exam covers a range of algebraic topics. Successful students will demonstrate conceptual understanding of the properties and operations of real and complex numbers. They will be able to generalize through the use of variables resulting in facility with algebraic expressions. They will solve single and systems of linear equations and inequalities and will be able to use them to represent contextual situations. Successful students also will be able to demonstrate facility with estimating and verifying solutions of various non-linear equations, making use of technology where appropriate to do so. Finally, students will demonstrate knowledge of functions and their properties – distinguishing among quadratic, higher degree polynomial, exponential, and piecewise-defined functions – and recognize and solve problems that can be modeled by these functions. They will be required to analyze the models, both symbolically and graphically, and to determine and effectively represent their solution(s). There are a variety of types of test items developed that will assess this content, including some that cut across the objectives in a standard and require students to make connections and, where appropriate, solve rich contextual problems.

There are sixty questions on the core Algebra II End-of-Course Exam including fifty multiple choice (1 point each), seven short answer (2 points each), and three extended response (4 points each). About one-third of the student's score will be based on the short-answer and extended-response items. The test is designed to take approximately 90 -120 minutes, comprised of two 45-60 minute sessions, one of which will allow calculator use. However, some students may require—and should be allowed—additional time to complete the test. Test items, in particular extended-response items, may address more than one content objective and benchmark within a standard. Each standard within the exam is assigned a priority, indicating the approximate percentage of points allocated to that standard on the test.

**Optional Algebra II End-of-Course Exam modules:** Optional modules in Data and Statistics, Probability, Logarithmic Functions, Trigonometric Functions, Matrices, Conic Sections, and Sequences and Series have been developed. One or more of these modules may be used by states, districts, or schools whose curriculum includes these topic areas in Algebra II or an equivalent course. Each module will include eight items: six multiple-choice, one short answer, and one extended response. Fifty percent of the student's score on the modules will be based on the combined scores of the short answer and extended response items. A student should be allowed approximately 20 minutes for each module. Calculators are allowed on all of the modules.

**Algebra II End-of-Course Exam calculator policy:** The appropriate and effective use of technology is an essential practice in the Algebra II classroom. At the same time students should learn to work mathematically without the use of technology. Computing mentally or with paper and pencil is required on the Algebra II End-of-Course Exam and should be expected in classrooms where students are working at the Algebra II level. It is therefore important that the Algebra II End-of-Course Exam reflect both practices. For purposes of the Algebra II End-of-Course Exam, students are expected to have access to a calculator for one of the two testing sessions, and use of a graphing calculator is strongly recommended. Scientific or four-function calculators are permitted but not recommended because they do not have graphing capabilities. Students should not use a calculator that is new or different for them on the exam but rather should use the calculator they are accustomed to and use every day in their classroom work. For more information about technology use on the Algebra II End-of-Course Exam see the calculator policy at the end of this document

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**Algebra II level curriculum:** Function modeling and problem solving is the heart of curriculum at the Algebra II level. Mathematical modeling consists of recognizing and clarifying mathematical structures that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution, and interpreting the solution in the context of the original problem. Students must be able to solve practical problems, representing and analyzing the situation using symbols, graphs, tables, or diagrams. They must effectively distinguish relevant from irrelevant information, identify missing information, acquire needed information, and decide whether an exact or approximate answer is called for, with attention paid to the appropriate level of precision. After solving a problem and interpreting the solution in terms of the context of the problem, they must check the reasonableness of the results and devise independent ways of verifying the results.

The standards included in this document are intended to reflect this curricular focus and to guide the work of the test designers and the test item developers. It is also the case that curriculum at the Algebra II level will include content and processes not included on the Algebra II End-of-Course Exam, as some are not easily assessed by a test of this nature. Problems that require extended time for solution should also be addressed in the Algebra II level classroom, even though they are not able to be included in this end-of-course exam.

**Algebra II level classroom practices:** Effective communication using the language of mathematics is essential in a class engaged in Algebra II level content. Correct use of mathematical definitions, notation, terminology, syntax, and logic should be required in all work at the Algebra II level. Students should be able to translate among and use multiple representations of functions fluidly and fluently. They should be able to report and justify their work and results effectively. To the degree possible, these elements of effective classroom practice are reflected in these Algebra II End-of-Course Exam content standards.

**The Algebra II End-of-Course Exam documents:** There are two versions of the Algebra II End-of-Course Exam, one containing the standards, objectives and benchmarks, and the other an expanded version of the same document. This expanded version of the Algebra II End-of-Course Exam Content Standards includes a second column to the right of the benchmarks with explanatory comments and examples. These comments and examples are meant to add clarity to the meaning of the benchmarks for teachers and test item writers. Examples are provided only when necessary for clarity and are not meant to be exhaustive or used as sample test items. In some instances, in the standards and in the explanations, the word "including" is used followed by a list. The word "including" does not translate to "all inclusive" but rather means "including but not limited to."

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**Core: Operations on Numbers and Expressions  
Priority: 15%**

Successful students will be able to perform operations with rational, real, and complex numbers, using both numeric and algebraic expressions, including expressions involving exponents and roots. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and, where appropriate, solve contextual problems.

Content Benchmarks	Explanatory Comments and Examples
<b>O1. Real numbers</b>	
a. Convert between and among radical and exponential forms of numerical expressions.	<ul style="list-style-type: none"> <li>Convert between expressions involving rational exponents and those involving roots and integral powers. <i>Example:</i> <math>5^{\frac{3}{2}} = \sqrt{5^3} = 5\sqrt{5}</math>; <math>\sqrt[4]{27} = \sqrt[4]{3^3} = 3^{\frac{3}{4}}</math>.</li> </ul>
b. Simplify and perform operations on radical numerical expressions.	<ul style="list-style-type: none"> <li>Convert radicals to alternate forms and use the understanding of this conversion to perform calculations with numbers in radical form. <i>Example:</i> <math>\sqrt{8} + \sqrt{18} = 2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}</math></li> </ul>
c. Apply the laws of exponents to numerical expressions with rational and negative exponents to order and rewrite them in alternative forms.	<ul style="list-style-type: none"> <li>Apply the properties of exponents in numeric expressions. <i>Example:</i> <math>3^5 \cdot 3^2 = 3^{(5+2)}</math>, <math>\frac{3^5}{3^2} = 3^{(5-2)}</math>, <math>(3^5)^2 = 3^{(5 \cdot 2)}</math> <math>3^{\frac{5}{2}} = (\sqrt{3})^5 = \sqrt{3^5} = 9\sqrt{3}</math></li> <li>Convert between expressions involving negative exponents and those involving only positive ones. <i>Example:</i> <math>3^{-2} = \frac{1}{9}</math>; <math>\frac{2^{-3}}{7^{-1}} = \frac{7}{2^3} = \frac{7}{8}</math></li> </ul>

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<b>O2. Complex numbers</b>	
<p>a. Represent complex numbers in the form <math>a+bi</math>, where <math>a</math> and <math>b</math> are real; simplify powers of pure imaginary numbers.</p>	<ul style="list-style-type: none"> <li>• Every real number, <math>a</math>, is a complex number because it can be expressed as <math>a+0i</math>.</li> <li>• Represent the square root of a negative number in the form <math>bi</math>, where <math>b</math> is real; simplify powers of pure imaginary numbers.</li> </ul> <p><i>Example:</i> <math>\sqrt{-8} = 2i\sqrt{2}</math></p> <p><i>Example:</i> <math>\sqrt{-256} = 16i</math></p> <p><i>Example:</i> <math>i^5 = -i</math></p>
<p>b. Perform operations on the set of complex numbers.</p>	<ul style="list-style-type: none"> <li>• Add, subtract, and multiply complex numbers using the rules of arithmetic.</li> <li>• Divide complex numbers using conjugates.</li> <li>• This process can also be applied to the division of irrational numbers involving square roots such as <math>a+\sqrt{b}</math> and <math>a-\sqrt{b}</math>.</li> </ul> <p><i>Example:</i> <math>\frac{(5+4i)}{3-2i} = \frac{(5+4i)(3+2i)}{(3-2i)(3+2i)} = \frac{15+22i+8i^2}{9-4i^2} = \frac{7+22i}{13}</math> or <math>\frac{7}{13} + \frac{22}{13}i</math></p>
<b>O3. Algebraic expressions</b>	
<p>a. Convert between and among radical and exponential forms of algebraic expressions.</p>	<p><i>Example:</i> <math>x^{\frac{3}{2}} = \sqrt{x^3} = x\sqrt{x}</math></p> <p><i>Example:</i> <math>\sqrt{a^6 b^4} = \left  a^{\frac{6}{2}} b^{\frac{4}{2}} \right  =  a^3  b^2</math></p>
<p>b. Simplify and perform operations on radical algebraic expressions.</p>	<p><i>Example:</i> <math>\sqrt{x^2 + 6x + 9} = \sqrt{(x+3)^2} =  x+3 </math></p>

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<p>c. Apply the laws of exponents to algebraic expressions, including those involving rational and negative exponents, to order and rewrite them in alternative forms.</p>	<p><i>Example:</i> <math>a^4 \cdot a^3 = a^{(4+3)}</math>, <math>\frac{a^4}{a^3} = a^{(4-3)}</math>, <math>(a^4)^3 = a^{(4 \cdot 3)}</math></p> <p><i>Example:</i> <math>y^{-2} = \frac{1}{y^2}</math>, <math>z^{\frac{2}{3}} = \sqrt[3]{z^2}</math></p> <p><i>Example:</i> <math>\frac{r^{-3}}{t^{-2}} = \frac{t^2}{r^3}</math>, <math>\frac{a^{-4}}{b^3} = \frac{1}{a^4 b^3}</math>, <math>(a^3 b^5)^2 = a^6 b^{10}</math></p>
<p>d. Perform operations on polynomial expressions.</p>	<ul style="list-style-type: none"> <li>• Limit to at most multiplication of a binomial by a trinomial.</li> <li>• For division limit the divisor to a linear or factorable quadratic polynomial.</li> <li>• Division may be performed using factoring.</li> </ul>
<p>e. Perform operations on rational expressions, including complex fractions.</p>	<ul style="list-style-type: none"> <li>• These expressions should be limited to linear and factorable quadratic denominators.</li> <li>• Complex fractions should be limited to simple fractions in numerators and denominators.</li> </ul> <p><i>Example:</i></p> $\frac{\left(\frac{1}{a} + \frac{1}{b}\right)}{\frac{(a+b)}{ab}} \div ab = \frac{(a+b)}{(b+a)} \div ab = (a+b) \cdot \frac{ab}{(b+a)} \div ab = ab \div ab = 1.$
<p>f. Identify or write equivalent algebraic expressions in one or more variables to extract information.</p>	<p><i>Example:</i> The expression, <math>C + 0.07C</math>, represents the cost of an item plus sales tax, while <math>1.07C</math> is an equivalent expression that can be used to simplify calculations of the total cost.</p> <p><i>Example:</i> <math>x - \frac{y^2}{x}</math> can be rewritten as <math>\frac{x^2 - y^2}{x}</math></p>

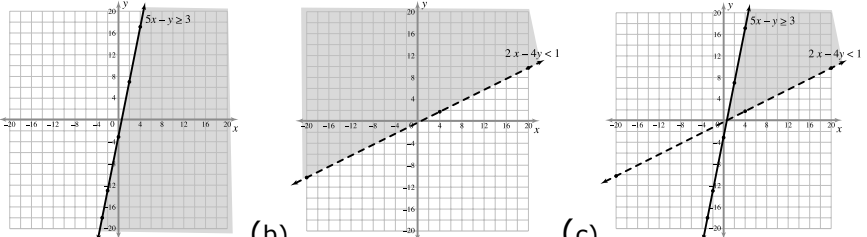
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**Core: Equations and Inequalities  
Priority: 20%**

Successful students will be able to solve and graph the solution sets of equations and inequalities and systems of linear equations and inequalities. The types of equations are to include linear and first order absolute value, quadratic, exponential, rational, radical, and higher order polynomials; the types of inequalities are to include linear and quadratic. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and, where appropriate, solve contextual problems. In contextual problems students will be required to graph and interpret their solutions in terms of the context. (Contextual test items will be limited to inequalities, systems of equations and inequalities, and those equations that do not represent a function.)

Content Benchmarks	Explanatory Comments and Examples
<b>E1. Linear equations and inequalities</b>	
a. Solve equations and inequalities involving the absolute value of a linear expression.	<i>Example:</i> $ x-6  \leq 8 \Rightarrow (x-6) \leq 8$ and $-(x-6) \leq 8$ $\Rightarrow x \leq 14$ and $x \geq -2$
b. Express and solve systems of linear equations in three variables with and without the use of technology.	<ul style="list-style-type: none"> <li>Systems in three variables should be limited to those with integer solutions and small integral coefficients.</li> </ul> <i>Example:</i> Solve the system $2x + z = 11$ $x - y + z = 6$ $x + 2y = 7$ OR $2x + 2y - z = 1$ $2x - 4y + z = 3$ $3x + 4y + 3z = 4$
c. Solve systems of linear inequalities in two variables and graph the solution set.	<i>Example:</i> Graph the solution set of the following system. $3x - 2y < 7, \quad x \geq 0, \quad y \leq 0$ <ul style="list-style-type: none"> <li>Recognize that the graphic solution of a linear inequality is either an open or closed half plane.</li> </ul> <i>Example:</i> Graphs (a) and (b) illustrate that the graph of a linear inequality is a half plane. Graph (c) illustrates a solution to the question: What is the set of points $(x, y)$ that satisfies both $5x - y \geq 3$ and $2x - 4y < 1$ ?

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<p>d. Recognize and solve problems that can be represented by single variable linear equations or inequalities or systems of linear equations or inequalities involving two or more variables. Interpret the solution(s) in terms of the context of the problem.</p>	<ul style="list-style-type: none"> <li>• This includes using and interpreting appropriate units of measurement and precision for the given application.</li> <li>• Common problems are those that involve, time/rate/distance, percentage increase or decrease, ratio and proportion; mixture problems and break-even problems.</li> </ul>
<b>E2. Nonlinear equations and inequalities</b>	
<p>a. Solve single-variable quadratic, exponential, rational, radical, and factorable higher-order polynomial equations over the set of real numbers, including quadratic equations involving absolute value.</p>	<ul style="list-style-type: none"> <li>• Use information gathered from a polynomial equation to determine the number and nature of its solutions:             <ul style="list-style-type: none"> <li>○ The number of solutions, real and complex, counted by multiplicity, for a polynomial is equal to its degree.</li> <li>○ Some polynomial equations, including quadratics, may have no real solutions.</li> <li>○ All complex solutions come in pairs (conjugates) making it necessary that all odd polynomial functions have at least one real solution.</li> </ul> </li> <li>• Explain the relationship between the number of real (and complex) solutions and the graph of a polynomial equation.</li> <li>• Solve equations and inequalities numerically, graphically, and algebraically, with and without the use of technology, making connections between solution strategies.</li> <li>▪ Solve power equations with integer exponents, <math>ax^n = b</math> algebraically, graphically, and using technology.             <p style="margin-left: 40px;"><i>Power equations can represent area or volume; polynomial equations can represent projectile height or profit maximization.</i></p> </li> <li>• Solve radical equations numerically, algebraically, and graphically,</li> </ul>

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	<p>with and without the use of technology.</p> <ul style="list-style-type: none"> <li>• Use the factored form of a polynomial to determine the real solutions.</li> <li>• Consider domain restrictions (asymptotes or undefined values) when finding solutions in rational and radical equations.</li> <li>• Know which operations on an equation may produce an equation with the same solutions, and which produce an equation with fewer or more solutions (lost or extraneous solutions).</li> </ul>
<p>b. Solve single variable quadratic equations and inequalities over the complex numbers; graph real solution sets on a number line.</p>	<ul style="list-style-type: none"> <li>• Solve quadratic equations and inequalities using factoring, completing the square, and the quadratic formula.</li> <li>• Use a calculator to approximate the solutions of a quadratic equation related to an inequality and as an aid in graphing.</li> <li>• Select and explain a method of solution (e.g., exact vs. approximate) that is effective and appropriate for a given problem.</li> <li>• Determine a single variable quadratic equation given its solutions.</li> <li>• Recognize that complex solutions come in conjugate pairs of the form <math>a + bi</math> and <math>a - bi</math>.</li> </ul>
<p>c. Use the discriminant, <math>D = b^2 - 4ac</math>, to determine the nature of the solutions of the equation <math>ax^2 + bx + c = 0</math>.</p>	<ul style="list-style-type: none"> <li>• Describe how the discriminant, <math>D = b^2 - 4ac</math>, indicates the nature of the solutions of the equation <math>ax^2 + bx + c = 0</math>. <i>The solutions are real and distinct if <math>D &gt; 0</math>; real and equal if <math>D = 0</math>; and complex if <math>D &lt; 0</math>.</i></li> </ul>
<p>d. Graph the solution set of a two-variable quadratic inequality in the coordinate plane.</p>	<ul style="list-style-type: none"> <li>• Students might also be asked to write or identify the quadratic inequality from the graph.</li> <li>• Recognize that complex solutions come in pairs, complex numbers in the forms <math>a + bi</math> and the conjugate <math>a - bi</math>.</li> </ul>
<p>e. Rewrite nonlinear equations and inequalities to express them in multiple forms in order to facilitate finding a solution set or to extract information about the relationships or graphs indicated.</p>	<ul style="list-style-type: none"> <li>• Inequalities and equations that do not represent functions may be included. (e.g. horizontal parabolas) <i>Example:</i> Explain how you know how to find the x-intercept for the equation <math>x = y^2 - 6y + 8</math>.</li> </ul>

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	<p><i>Example:</i> For <math>y+1 \geq (x-3)^2</math> find the vertex and sketch a graph.</p> <ul style="list-style-type: none"><li>• Inequalities and equations in one variable will be included.</li></ul> <p><i>Example:</i> <math>x^2 + 8x + 7 = 0 \rightarrow (x+7)(x+1) = 0 \rightarrow x = -7</math> or <math>x = -1</math></p> <ul style="list-style-type: none"><li>• Rewriting or solving a formula in several variables for one variable in terms of the others will be included.</li></ul> <p><i>Example:</i> Solve <math>V = \frac{1}{3}\pi r^2 h</math> for <math>r</math> or <math>h</math>.</p> <p><i>Example:</i> Solve <math>A = \pi r^2 + 2\pi rh</math> for <math>r</math> or <math>h</math>.</p>
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**Core: Polynomial and Rational Functions  
Priority: 30%**

Successful students will be able to use tables, graphs, verbal statements and symbols to represent and analyze quadratic, rational, and higher order polynomial functions. They will be able to recognize and solve problems that can be modeled using these functions. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and solve rich contextual problems.

Content Benchmarks	Explanatory Comments and Examples
<b>P1. Quadratic functions</b>	
<p>a. Determine key characteristics of quadratic functions and their graphs.</p>	<ul style="list-style-type: none"> <li>• Key characteristics include domain and range, vertex, minimum/maximum, intercepts, axis of symmetry, and end behavior.</li> <li>• Recognize that except when <math>a = 0</math>, the graph of <math>f(x) = ax^2 + bx + c</math> is a parabolic curve that always crosses the <math>y</math>-axis but may or may not cross the <math>x</math>-axis; that it opens up when <math>a &gt; 0</math> and down when <math>a &lt; 0</math>.</li> <li>• Recognize the relationship between the intercepts and the factors of a quadratic function.</li> <li>• Recognize <math>x = \frac{-b}{2a}</math> as a way of determining the <math>x</math>-coordinate of the vertex for a parabola in the form <math>y = ax^2 + bx + c</math>.</li> <li>• Recognize the relationship between complex solutions of a quadratic equation and the characteristics and position of its graph.</li> </ul> <p><i>Example:</i> Use different forms of the function to extract information:  <math>y = x^2 - 6x + 8</math> makes the <math>y</math>-intercept obvious,  <math>y = (x - 2)(x - 4)</math> provides access to the zeros, and  <math>y = (x - 3)^2 - 1</math> makes it easy to find the vertex and sketch the graph.</p>

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<p>b. Represent quadratic functions using tables, graphs, verbal statements, and equations. Translate among these representations.</p>	
<p>c. Describe the effect that changes in the parameters of a quadratic function have on the shape and position of its graph.</p>	<ul style="list-style-type: none"> <li>• Transform the parent quadratic function, <math>y = x^2</math>, to translate, reflect, rotate, stretch or compress the graph.</li> <li>• Investigate the changes that occur when the coefficients and constant are changed in a quadratic function in the form, <math>f(x) = ax^2 + bx + c</math>.</li> </ul>
<p>d. Recognize, express, and solve problems that can be modeled using quadratic functions. Interpret their solutions in terms of the context.</p>	<ul style="list-style-type: none"> <li>• Recognize situations, in which quadratic models are appropriate; create and interpret quadratic models to answer questions about those situations.</li> <li>• This includes using and interpreting appropriate units of measurement and precision for the given application.</li> </ul> <p><i>Example:</i> Determine the height of an object above the ground <math>t</math> seconds after it has been thrown upward at an initial velocity of <math>v_0</math> feet per second from a platform <math>d</math> feet above the ground.</p> <p><i>Example:</i> Determine the relationship between the length of a side of a cube and its surface area.</p>
<b>P2. Higher-order polynomial and rational functions</b>	
<p>a. Determine key characteristics of power functions in the form <math>f(x) = ax^n</math> <math>a \neq 0</math>, for positive integral values of <math>n</math> and their graphs.</p>	<ul style="list-style-type: none"> <li>• Recognize the basic power functions as the parent functions for many types of polynomials.</li> </ul> <p><i>Example:</i> Determine the parent function for all cubic equations in the form <math>y = ax^3 + bx^2 + cx + d</math>, where <math>a \neq 0</math>,</p> <ul style="list-style-type: none"> <li>• Identify the type of symmetry and relate to odd/even exponents.</li> </ul>
<p>b. Determine key characteristics of polynomial functions and their graphs.</p>	<ul style="list-style-type: none"> <li>• Key characteristics include domain and range, intercepts, end behavior, and degree. <i>Know that polynomial functions of degree <math>n</math> have the general form <math>f(x) = ax^n + bx^{n-1} + \dots + px^2 + qx + r</math> where <math>n</math> is a positive integer. The degree of the polynomial function is the</i></li> </ul>

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	<p><i>largest power of its terms for which the coefficient is non-zero. The leading coefficient is the coefficient of the term of highest degree.</i></p> <ul style="list-style-type: none"> <li>○ Identify and use the degree of a polynomial or its factors to interpret characteristics of the function or its graph.</li> <li>○ Note that every polynomial function of odd degree has at least one real zero.</li> <li>○ Understand the concept of the <i>multiplicity</i> of a root of a polynomial equation and its relationship to the graph of the related polynomial function.</li> </ul>
<p>c. Represent polynomial functions using tables, graphs, verbal statements, and equations. Translate among these representations.</p>	<ul style="list-style-type: none"> <li>• Identify or write a polynomial function of a given degree.</li> <li>• Decide if a given graph or table of values suggests a higher order polynomial function.</li> </ul>
<p>d. Determine key characteristics of simple rational functions and their graphs.</p>	<ul style="list-style-type: none"> <li>• Key characteristics include domain and range, intercepts, types of symmetry, and asymptotic and end behavior.</li> <li>• Simple rational functions are those with linear, quadratic, or monomial denominators, including power functions of the form <math>f(x) = ax^n</math> (<math>a \neq 0</math>) for negative integral values of <math>n</math>.</li> </ul>
<p>e. Represent simple rational functions using tables, graphs, verbal statements, and equations. Translate among these representations.</p>	<ul style="list-style-type: none"> <li>• Simple rational functions are those with linear, quadratic, or monomial denominators, including power functions of the form <math>f(x) = ax^n</math> (<math>a \neq 0</math>) for negative integral values of <math>n</math>.</li> </ul>
<p>f. Recognize, express, and solve problems that can be modeled using polynomial and simple rational functions. Interpret their solutions in terms of the context.</p>	<ul style="list-style-type: none"> <li>• This includes using and interpreting appropriate units of measurement and precision for the given application.</li> </ul> <p><i>Example:</i> The volume of a cylinder whose radius is 5 more than twice the radius, <math>r</math>, of a given sphere and whose height is twice its own radius is <math>6750\pi</math>. Determine the radius of the given sphere.</p>

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**Core: Exponential Functions  
Priority: 20%**

Successful students will be able to use tables, graphs, verbal statements and symbols to represent, analyze, model, and interpret graphs of exponential functions. While some facility with the properties of logarithms may be helpful it is not required on the core exam. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and solve rich contextual problems.

Content Benchmarks	Explanatory Comments and Examples
<b>X1. Exponential functions</b>	
a. Determine key characteristics of exponential functions and their graphs.	<ul style="list-style-type: none"> <li>Key characteristics include domain and range, intercepts, and end behavior.</li> </ul>
b. Represent exponential functions using tables, graphs, verbal statements, and equations. Represent exponential expressions in multiple forms. Translate among these representations.	<ul style="list-style-type: none"> <li>Know that exponential functions have the general form <math>f(x) = ab^x + c</math>, <math>b &gt; 0</math>, and <math>b \neq 1</math>.</li> <li>Distinguish between and graph exponential functions that are growth functions such as <math>f(x) = an^x</math> (<math>a &gt; 0</math>, <math>x &gt; 0</math>, and <math>n &gt; 1</math>) or decay functions such as <math>f(x) = an^{-x}</math> (<math>a &gt; 0</math>, <math>x &gt; 0</math>, and <math>n &gt; 1</math>).</li> <li>Decide if a given graph or table of values represents an exponential function. <i>Be aware that it can be very difficult to distinguish exponential graphs from graphs of other functions, particularly polynomial functions, over small regions or particular subsets of their domains. Sometimes the context of an underlying situation can suggest a likely type of function model.</i></li> <li>Translate from exponential to logarithmic form and vice versa.</li> </ul>
c. Describe the effect that changes in a parameter of an exponential function have on the shape and position of its graph.	<ul style="list-style-type: none"> <li>Explain or illustrate the effect that changes in a parameter (<math>a</math> or <math>c</math>) or the base (<math>b</math>) have on the graph of the exponential function <math>f(x) = ab^x + c</math>.</li> </ul>
d. Recognize, express, and solve problems that can be modeled using exponential functions, including those where logarithms provide an efficient method of solution. Interpret their solutions in terms of the context.	<ul style="list-style-type: none"> <li>A logarithm is an exponent that depends on the base used. Logarithms provide an efficient method for solving problems with variable exponents. Limitation: Logarithms may be used to solve problems but will not be used in the text of an item. <i>If <math>x^y = z</math>, <math>x &gt; 0</math>, <math>x \neq 1</math>, and <math>z &gt; 0</math>, then <math>y</math> is the logarithm to</i></li> </ul>

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	<p style="text-align: center;"><i>the base <math>x</math> of <math>z</math>. The equation <math>y = \log_x z</math> is one of three equivalent forms of expressing the relation <math>x^y = z</math> (the other being <math>x = \sqrt[y]{z}</math>.</i></p> <ul style="list-style-type: none"><li>• Exponential problems in which the student must determine the exponent may also be solved using graphing techniques.</li><li>• Exponential growth functions using base <math>e</math> may be included (e.g. . <math>A = Pe^{rt}</math>, or <math>y = Ce^{kt}</math>). Formulas will be provided in these cases.</li><li>• This includes using and interpreting appropriate units of measurement and precision for the given application.</li></ul> <p><i>Example:</i> If a culture had 500 cells at noon and 600 cells at 1:00 PM, what is the approximate doubling time of the cell population? Approximately how many cells will there be at 4:00 PM?</p> <p><i>Example:</i> If radioactive iodine 123 has a half life of 8 days, what percentage of an original dose will remain in the patient's thyroid twelve hours after a medical test has been performed in which iodine 123 was administered?</p>
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**Core: Function Operations and Inverses  
Priority: 15%**

Successful students will be able to perform function operations of addition, subtraction, multiplication, division, and composition and to combine several functions defined over restricted domains to form a piecewise-defined function. They will be able to determine, graph and analyze the inverse of a function and use composition to determine whether two functions are inverses. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections.

Content Benchmarks	Explanatory Comments and Examples
<b>F1. Function operations</b>	
<p>a. Combine functions by addition, subtraction, multiplication, and division.</p>	<p><i>Example:</i> If <math>f(x) = 3x^4 - 5x^3 + 3</math>, and <math>g(x) = x^4 - 3x^3 + 2x^2 + 5</math>, then <math>f(x) - g(x) =</math>  <math>(3x^4 - 5x^3 + 3) - (x^4 - 3x^3 + 2x^2 + 5)</math>  <math>= (3x^4 - x^4) + (-5x^3 - (-3x^3)) - 2x^2 + (3 - 5)</math>  <math>= 2x^4 - 2x^3 - 2x^2 - 2</math></p> <p><i>Example:</i> If <math>h(x) = x^4 - 16</math> and <math>j(x) = x - 2</math>, then <math>h(x) \div j(x) =</math>  <math>(x^4 - 16) \div (x - 2) = \frac{(x^2 + 4)(x^2 - 4)}{x - 2} = \frac{(x - 2)(x + 2)(x^2 + 4)}{x - 2} =</math>  <math>(x + 2)(x^2 + 4)</math></p> <p><i>Example:</i> If <math>r(x) = x^2 + 3x - 1</math> and <math>s(x) = x + 3</math>, then show that <math>r(x) \cdot s(x) =</math>  <math>(x^2 + 3x - 1)(x + 3) = x^3 + 6x^2 + 8x - 3</math></p>
<p>b. Determine the composition of two functions, including any necessary restrictions on the domain.</p>	<p><i>Example:</i> If <math>f(x) = 3x - 2</math>, and <math>g(x) = \sqrt{x + 1}</math> find <math>f(g(x))</math> and <math>g(f(x))</math>. Include domain restrictions on each.</p> <p><i>Example:</i> If <math>f(x) = \frac{3}{5} + \frac{2}{5x}</math>, and <math>g(x) = \frac{2}{5x - 3}</math>, determine whether or not <math>f</math> and <math>g</math> are inverses and explain how you know.</p>

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<b>F2. Inverse functions</b>	
a. Describe the conditions under which an inverse relation is a function.	<ul style="list-style-type: none"> <li>Consider graphic conditions for an inverse relation to be a function.</li> <li>Use the horizontal line test to determine whether a function's inverse will be a function. Consider domain restrictions for existence of an inverse function.</li> <li>Recognize that the inverse of a quadratic function is a function only when its domain is restricted.</li> </ul>
b. Determine and graph the inverse relation of a function.	<ul style="list-style-type: none"> <li>Include inverses which may not be functions.</li> </ul> <p><i>Example:</i> If <math>y = x^2</math>, the inverse is <math>x = y^2</math>, which can also be written as <math>y = \pm\sqrt{x}</math>. Explain why the inverse function would be only the positive part of the graph.</p> <ul style="list-style-type: none"> <li>Explain why the graph of a function and its inverse, are reflections of each other over the line <math>y = x</math>.</li> <li>Show that when the inverse of a function is a function <math>f^{-1}(f(x)) = x</math> and <math>f(f^{-1}(x)) = x</math></li> </ul> <p><i>Example:</i> Determine <math>f^{-1}(x)</math> when <math>f(x) = 3x - 5</math>. <i>Example:</i> Determine <math>g(x)</math> when <math>g^{-1}(x) = 9x^2 + 81</math>.</p> <ul style="list-style-type: none"> <li>Inverses of exponential functions may be required graphically but not in algebraic form.</li> </ul> <p><i>Example:</i> Graphically determine <math>f^{-1}(x)</math> when <math>f(x) = 2^x - 7</math>.</p>
<b>F3. Piecewise-defined functions</b>	
a. Determine key characteristics of absolute value, step, and other piecewise-defined functions.	<ul style="list-style-type: none"> <li>Key characteristics include vertices, intercepts, end behavior, slope of linear sections, and discontinuities.</li> <li>Determine the vertex, slope of each branch, intercepts, and end behavior of an absolute value graph.</li> </ul>

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<p>b. Represent piecewise-defined functions using tables, graphs, verbal statements, and equations. Translate among these representations.</p>	<ul style="list-style-type: none"><li>• Interpret the algebraic representation of a piecewise defined function; graph it over the appropriate domain.</li><li>• Write an algebraic representation for a given piecewise defined function.</li></ul>
<p>c. Recognize, express, and solve problems that can be modeled using absolute value, step, and other piecewise-defined functions. Interpret their solutions in terms of the context.</p>	<ul style="list-style-type: none"><li>• This includes using and interpreting appropriate units of measurement and precision for the given application.</li><li>• Applications may include postage rates, salary increases, or cellular telephone charges.</li></ul>

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**Module: Data and Statistics**

Successful students will be able to analyze, interpret, compare, and compute with summary statistics for sets of data. Analysis of bivariate data includes the determination and interpretation of regression lines and correlation coefficients. While some important components in the study of data and statistics, such as sampling techniques, question formulation, and experiment design are addressed when possible on this module of the Algebra II End-of-Course Exam, those topics will be expected to be assessed in more depth in the classroom. This module includes a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and solve rich contextual problems.

Content Benchmarks	Explanatory Comments and Examples
<b>S1. Summarization and comparison of data sets</b>	
<p>a. Summarize and compare data sets using statistical methods.</p>	<ul style="list-style-type: none"> <li>• For univariate categorical data, use percentages and proportions (relative frequencies).</li> <li>• For bivariate categorical data, use conditional (row or column) percentages or proportions.</li> <li>• For univariate quantitative data, use measures of center (mean and median) and measures of spread (percentiles, quartiles interquartile range, and standard deviation).</li> <li>• Graphically represent measures of center and spread (variability) for quantitative data.</li> <li>• Use box plots to compare key features of data distributions.</li> </ul>
<p>b. Determine, use, and identify potential misuses of weighted averages.</p>	<ul style="list-style-type: none"> <li>• Identify and interpret common instances of weighted averages.</li> <li>• Analyze variation in weighted averages and distinguish change due to weighting from changes in the quantities measured.</li> <li>• Contexts may include grade averages, stock market indexes, consumer price index, or unemployment rate.</li> </ul> <p><i>Example:</i> Suppose a company employed 100 women with average annual salaries of \$20,000 and 500 men with average salaries of \$40,000. After a</p>

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	<p>change in management, they employed 200 women and 400 men. To correct past inequities, the new management increased women's salaries by 25% and men's salaries by 5%. Despite these increases, the company's average salary declined by almost 1%.</p>
<p>c. Use a computer or calculator to find a linear regression equation (least squares line) as a model for data that suggest a linear trend, and determine the correlation coefficients.</p>	<ul style="list-style-type: none"> <li>• Regression can be used to summarize bivariate quantitative data.</li> <li>• Know that the least squares line passes through the point <math>(\bar{x}, \bar{y})</math>, where the mean of the x-coordinates of the data points is <math>\bar{x}</math> and the mean of the y-coordinates of the data points is <math>\bar{y}</math>.</li> <li>• Recognize correlation as a number between <math>-1</math> and <math>+1</math> that measures the direction and strength of linear association between two variables.</li> </ul>

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<b>S2. Interpretation and communication through data</b>	
<p>a. Analyze the strength of the linear relationship indicated by the regression line.</p>	<ul style="list-style-type: none"> <li>• Common terms such as least squares line, regression line, and line of best or good fit will be used in the problems.</li> <li>• Use the relationship among the standard deviation, correlation coefficient, and slope of the regression line to assess the strength of linear association suggested by an underlying scatter plot.</li> <li>• Interpret the slope of a linear trend line in terms of the data being studied.</li> <li>• Identify the effect of outliers on the position and slope of the regression line.</li> <li>• Analyze a residual plot to informally assess whether a line provides a good model for a set of data-</li> </ul>
<p>b. Interpret data and communicate conclusions effectively.</p>	<ul style="list-style-type: none"> <li>• Interpret, interpolate, and judiciously extrapolate from graphs and tables.</li> <li>• Use the basic properties of confidence intervals to make simple predictions and answer questions about statistical data.</li> <li>• State conclusions in terms of the question(s) being investigated.</li> <li>• Use appropriate statistical language when reporting on plausible answers that go beyond the data actually observed.</li> <li>• Interpret oral, written, graphic, pictorial, or multi-media reports on data.</li> </ul>
<p>c. Make judgments regarding accuracy, reasonableness, and bias in the uses of data.</p>	<ul style="list-style-type: none"> <li>• Identify and explain misleading uses of data by considering the completeness and source of the data, the design of the study, and the way the data are analyzed and displayed.</li> <li>• Recognize the difference between correlation or association and causation.</li> </ul> <p><i>Example:</i> Determine whether the height or area of a bar graph is being used to represent the data; evaluate whether the scales of a graph are consistent</p>

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	and appropriate or whether they are being adjusted to alter the perception conveyed by the information.
d. Critique and justify various methods of sampling and data collection used in real world problems.	<ul style="list-style-type: none"><li>• Some possible real world applications may be clinical trials in medicine, an opinion poll, or a report on the effect of smoking on health.</li><li>• Distinguish between random sampling from a population in sample surveys and random assignment of treatments to experimental units in an experiment: <i>Random sampling is how items are selected from a population so that the sample data can be used to estimate characteristics of the population; random assignment is how treatments are assigned to experimental units so that comparisons among the treatment groups can allow cause and effect conclusions to be made.</i></li></ul>

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<b>Module: Probability</b>	
<p>Successful students will be able to quantify the likelihood that an event will occur through combinatorics and other counting principles, relative frequency, distributions, and the comparison of theoretical probability to simulations. Also included are binomial expansion and the relationship to Pascal’s triangle and binomial distributions. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and solve rich contextual problems.</p>	
Content Benchmarks	Explanatory Comments and Examples
<b>R1. Permutations, combinations, and probability</b>	
<p>a. Determine the number of ways events can occur using permutations, combinations, and other systematic counting methods.</p>	<ul style="list-style-type: none"> <li>• Know and apply organized counting techniques such as the Fundamental Counting Principle. <i>The Fundamental Counting Principle is a way of determining the number of ways a sequence of events can take place. If there are <math>n</math> ways of choosing one thing and <math>m</math> ways of choosing a second after the first has been chosen, then the Fundamental Counting Principle says that the total number of choice patterns is <math>n \times m</math>.</i></li> <li>• Simplify the factorial form of the permutation and combination formula to aid computation.</li> <li>• Distinguish between situations where replacement is not permitted and situations that permit replacement, including those involving circular arrangements.</li> <li>• Distinguish between situations where order matters and situations where it does not; select and apply appropriate means of computing the number of possible arrangements of the items in each case.</li> <li>• Use permutations and combinations to determine the number of possible outcomes of an event.</li> <li>• Use factorials to define combinations and permutations and to determine the number of ways an event can occur.</li> </ul> <p><i>Example:</i> Interpret <math>6!</math> as the product <math>6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1</math>; Show that <math>\frac{15!}{12!} = 15 \cdot 14 \cdot 13 = 2730</math>.</p> <p><i>Example:</i> Determine the number of different license plates that can be</p>

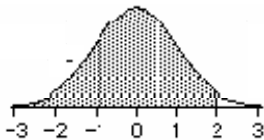
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	<p>formed with two letters and three numerals; determine how many are possible if all the letters had to come first; determine how many different subsets are possible for a set having six elements?</p> <p><i>Example:</i> Determine the number of different four-digit numbers that can be formed if the first digit must be non-zero and each digit may be used only once; determine how many are possible if the first digit must be non-zero but digits can be used any number of times.</p> <p><i>Example:</i> Determine the number of ways that first, second, and third place could be awarded in a race having 9 contestants if there are no ties <math>{}_9P_3</math>; determine the number of distinct committees of five people that can be formed from a class of 26 students <math>{}_{26}C_5</math>.</p>
<p>b. Relate the expansion of <math>(x + y)^n</math> (i.e., the binomial theorem) with the possible outcomes of a binomial experiment and/or the nth row of Pascal's triangle.</p>	
<p>c. Apply probability concepts to calculate the probability of events and to make informed decisions in practical situations.</p>	<ul style="list-style-type: none"> <li>• Construct and use decision trees. <i>A tree is a connected graph containing no closed loops (cycles).</i> <i>Note: Tree diagrams can also be used to analyze games such as tic-tac-toe or Nim or to simply organize outcomes.</i></li> <li>• Use the concepts of conditional probability to calculate simple probabilities.</li> <li>• Represent and analyze probabilities of independent events (e.g., repeated tossing of a coin, or throwing dice).</li> </ul>

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<p>d. Analyze and interpret actual data to estimate probabilities and predict outcomes, including those involving relative frequency.</p>	<ul style="list-style-type: none"> <li>• Recognize and use relative frequency as an estimate for probability.</li> <li>• If an action is repeated <math>n</math> times and a certain event occurs <math>b</math> times, the ratio <math>\frac{b}{n}</math> is called the relative frequency of the event occurring.</li> </ul> <p><i>Example:</i> In a sample of 100 randomly selected students, 37 of them could identify the difference in two brands of soft drinks. Based on these data, what is the best estimate of how many of the 2352 students in the school could distinguish between the soft drinks?</p>
<p>e. Compare theoretical probabilities with the results of simple experiments (<i>e.g.</i>, tossing number cubes, flipping coins, spinning spinners).</p>	<ul style="list-style-type: none"> <li>• Explain how the Law of Large Numbers defines the relationship between experimental and theoretical probabilities. <i>The law of large numbers indicates that if an event of probability <math>p</math> is observed repeatedly during independent repetitions, the ratio of the observed frequency of that event to the total number of repetitions approaches <math>p</math> as the number of repetitions becomes arbitrarily large.</i></li> <li>• Use simulations to estimate probabilities.</li> <li>• Use theoretical probability to determine the most likely result if an experiment is repeated a large number of times.</li> </ul>
<p>f. Compute and graph cumulative frequencies.</p>	

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R2. Probability distributions	
<p>a. Identify and distinguish between discrete and continuous probability distributions.</p>	<p><i>Example:</i> Compare and contrast the binomial distribution (discrete) and the normal distribution (continuous).</p>
<p>b. Identify the principal characteristics of the normal distribution and use them to estimate probabilities.</p>	<ul style="list-style-type: none"> <li>• Understand common examples that fit the normal distribution (e.g., height, weight) and examples that do not (salaries, housing prices, size of cities) and explain the distinguishing characteristics of each.</li> </ul> <p><i>The normal (or Gaussian) distribution is actually a family of mathematical distributions that are symmetric in shape with scores more concentrated in the middle than in the tails. They are sometimes described as bell shaped. Normal distributions may have differing centers (means) and scale (standard deviation). The standard normal distribution is the normal distribution with a mean of zero and a standard deviation of one. In this distribution, approximately 68% of the data lie within one standard deviation of the mean and 95% within two.</i></p> <div style="text-align: right;">  </div>
<p>c. Identify and describe the key characteristics of and create frequency distributions of both discrete and continuous data.</p>	<ul style="list-style-type: none"> <li>• Describe key characteristics (e.g., shape, symmetry/skewness, typical value, and/or spread) of a frequency distribution. <ul style="list-style-type: none"> <li><i>A frequency distribution shows the number of observations falling into each of several ranges of values; if the percentage of observations is shown, the distribution is called a relative frequency distribution. Both frequency and relative frequency distributions are portrayed through tables, histograms, or broken-line graphs.</i></li> </ul> </li> <li>• Use a probability distribution to assess the likelihood of the occurrence of an event</li> </ul>

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**Module: Logarithmic Functions**

Successful students will be able to define, represent, and model using logarithmic functions. Recognition of the inverse relationship between logarithmic and exponential functions is essential to this concept. They will apply the laws of logarithms, solve logarithmic equations, and use logarithms to solve exponential equations. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and solve rich contextual problems.

Content Benchmarks	Explanatory Comments and Examples
<b>L1. Logarithmic expressions and equations</b>	
a. Apply the properties of logarithms and use them to manipulate logarithmic expressions.	<ul style="list-style-type: none"> <li>• Represent logarithmic expressions in exponential form and exponential expressions in logarithmic form.</li> <li>• Understand that a logarithm is an exponent that depends on the base used.</li> <li>• The properties of logarithms include those related to powers, products, quotients, and changing the base.</li> </ul>
b. Solve logarithmic equations, paying attention to the possibility of extraneous roots.	<p><i>Example:</i> If <math>\log(x-3) + \log(x+2) = \log(14)</math>, then</p> $\log(x^2 - x - 6) = \log(14)$ $x^2 - x - 6 = 14$ $x^2 - x - 20 = 0$ $x = 5 \text{ or } -4$ <p>However, all solutions must be greater than 3 because of domain restrictions on the original logarithm functions, and the only solution is <math>x = 5</math>.</p>

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L2. Logarithmic functions	
a. Determine key characteristics of logarithmic functions.	<ul style="list-style-type: none"> <li>• Key characteristics include domain, range, and end behavior.</li> </ul>
b. Represent logarithmic functions using tables, graphs, verbal statements, and equations. Translate among these representations.	
c. Describe the effect that changes in the parameters of a logarithmic function have on the shape and position of its graph.	
d. Recognize, express, and solve problems that can be modeled using logarithmic functions. Interpret their solutions in terms of the context of the problem.	<ul style="list-style-type: none"> <li>• Use the properties of logarithms to solve problems involving logarithmic and exponential functions, including problems of growth and decay, chemical PH, and sound or earthquake intensity.</li> <li>• This includes using and interpreting appropriate units of measurement and precision for the given application.</li> </ul>

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**Module: Trigonometric Functions**

Successful students will be able to recognize and model periodic phenomena using trigonometric functions. They will understand the relationship among the unit circle, the geometric definitions of sine and cosine, and the degree and radian measures of angles and will apply this understanding to graph trigonometric functions, determine key characteristics of the functions and their graphs, and describe the effect of transformations on both the symbolic and graphical representations of the functions. There are a variety of types of assessment items including some that cut across the objectives in this standard and require students to make connections and solve rich contextual problems.

Content Benchmarks	Explanatory Comments and Examples
<b>T1. Trigonometric Functions</b>	
a. Recognize periodic phenomena and determine key characteristics of such phenomena.	<ul style="list-style-type: none"> <li>• Key characteristics include the period between repetitions, the frequency of the repetitions, and the range of values.</li> <li>• Periodic phenomena include sound waves, length of daylight, and situations involving circular motion.</li> </ul>
b. Use the relationship of the sine and cosine functions to a central angle of the unit circle to determine the exact trigonometric ratio of angles on the unit circle. ( $0^\circ$ to $360^\circ$ , $0$ to $2\pi$ )	<ul style="list-style-type: none"> <li>• Interpret the sine, cosine, and tangent functions corresponding to a central angle of the unit circle in terms of horizontal and vertical sides of right triangles based on that central angle.</li> </ul>
c. Explain and use both degree and radian measure for angles.	<ul style="list-style-type: none"> <li>• Convert between the degree and radian measure of an angle.</li> </ul>

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<p>d. Represent trigonometric functions using tables, graphs, verbal statements, and equations. Translate among these representations.</p>	<ul style="list-style-type: none"> <li>• Use the unit circle to extend the domain of the sine and cosine function to the set of real numbers.</li> </ul>
<p>e. Determine key characteristics of trigonometric functions and their graphs.</p>	<ul style="list-style-type: none"> <li>• Key characteristics include zeros, amplitude, period, phase shift, vertical shift, and asymptotes.</li> </ul>
<p>f. Describe the effect that changes in the parameters of an equation of a trigonometric function in the form, <math>f(x) = A\sin B(x - C) + D</math> (or the similar cosine function) have on the shape and position of its graph.</p>	<ul style="list-style-type: none"> <li>• Identify changes in the period, amplitude, vertical shift, and phase shift for the function.</li> <li>• Identify the zeros of a function that has a vertical shift of 0.</li> <li>• Identify and describe the transformations (vertical shifts, phase shifts, stretches and compressions) that occur when parameters are changed in trigonometric functions.</li> </ul>
<p>g. Recognize, express, and solve problems that can be modeled using trigonometric or other periodic functions.</p>	<p><i>Example:</i> Explain the difference between frequency modulation used in FM radio signals and amplitude modulation used in AM radio signals</p> <ul style="list-style-type: none"> <li>• Use Pythagorean identities for sine, cosine, and tangent to solve trigonometric equations and problems.</li> <li>• Use inverse trigonometric functions to solve equations and problems.</li> <li>• Solve problems in context that rely on trigonometric relationships.</li> <li>• This includes using and interpreting appropriate units of measurement and precision for the given application.</li> </ul>

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**Module: Matrices**

Successful students will be able to compute with and use matrices to organize information, solve systems of equations, and perform transformations of geometric figures. They will use and interpret matrix notation to represent a vector and perform operations on vectors and matrices. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and solve contextual problems.

Content Benchmarks	Explanatory Comments and Examples
<b>M1. Matrix arithmetic</b>	
a. Perform addition, subtraction, and scalar multiplication of matrices.	<ul style="list-style-type: none"> <li>• Recognize that matrix addition is associative and commutative.</li> <li>• Know and fluently use matrix notation for rows, columns, and entries of cells.</li> </ul>
b. Perform matrix multiplication.	<ul style="list-style-type: none"> <li>• Describe the characteristics of matrices that can be multiplied and those that cannot.</li> <li>• Recognize that matrix multiplication is not commutative and provide examples of square matrices <math>A</math> and <math>B</math> such that <math>AB \neq BA</math>.</li> <li>• Apply the associative property of matrix multiplication <i>The associative property of matrix multiplication states that if there are three matrices, <math>A</math>, <math>B</math>, and <math>C</math> such that <math>AB</math> and <math>BC</math> are defined, then <math>(AB)C</math> and <math>A(BC)</math> are defined and <math>(AB)C = A(BC)</math>.</i></li> <li>• Know and use the algorithm for matrix multiplication.</li> </ul>

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<b>M2. Solving systems of equations using matrices</b>	
a. Find the determinant of a 2x2 or 3x3 matrix.	
b. Determine the inverse of a 2x2 or 3x3 matrix or indicate that no inverse exists.	<ul style="list-style-type: none"> <li>• Know and use the definition and properties of the identity matrix.</li> <li>• Find the inverse of a 2x2 matrix if the inverse exists.</li> <li>• Use row reduction to find the inverse of a 3x3 matrix when it exists.</li> </ul>
c. Represent 2-variable and 3-variable systems of linear equations using matrices and use them to solve the system.	<ul style="list-style-type: none"> <li>• Translate between equation and matrix notation of a system of linear equations.</li> <li>• Use both technology and by-hand methods, including row reduction, substitution, and elimination, for systems involving 3 equations.</li> <li>• 3x3 systems to be solved by hand should have integer or simple fractional solutions and small integral coefficients.</li> </ul>
d. Solve a matrix equation.	<ul style="list-style-type: none"> <li>• Use the inverse of a matrix to solve a matrix equation.</li> </ul>

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<b>M3. Matrix transformations</b>	
<p>a. Use matrix tools to represent and transform geometric objects in the coordinate plane.</p>	<ul style="list-style-type: none"> <li>Use 2x2 matrices to represent transformations in the coordinate plane.</li> </ul> <p><i>Example:</i> <math>\begin{bmatrix} -1 &amp; 0 \\ 0 &amp; -1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}</math>  represents the rotation of the point <math>A(3, 1)</math> 180° about the origin.</p> <p><i>Example:</i> <math>\begin{bmatrix} -1 &amp; 0 \\ 0 &amp; 1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}</math>  represents the reflection of the point <math>A(3, 1)</math> over the <math>y</math>-axis.</p>
<b>M4. Vectors</b>	
<p>a. Represent vectors as matrices in two dimensions.</p>	<ul style="list-style-type: none"> <li>Use either horizontal or vertical representation of vectors, with an understanding of both.</li> <li>Use vectors to represent and transform geometric objects in the coordinate plane.</li> <li>Know the relationship between the ordered pair representation of a vector and its graphical representation.</li> </ul>
<p>b. Add, subtract, and compute the dot product of two-dimensional vectors; multiply a two-dimensional vector by a scalar.</p>	<ul style="list-style-type: none"> <li>Represent addition, subtraction, and scalar multiplication of vectors graphically.</li> <li>Use the dot product of two vectors to determine whether they are normal (perpendicular) to each other.</li> </ul>

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<b>Module: Conic Sections</b>	
<p>Successful students will be able to represent, analyze, and model using the circle, ellipse, and hyperbola. In addition parabolas with a horizontal axis are included. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and solve rich contextual problems.</p>	
Content Benchmarks	Explanatory Comments and Examples
<b>C1. Conic Sections</b>	
<p>a. Identify a parabola, circle, ellipse, or hyperbola from its equation, description, or key characteristics.</p>	<ul style="list-style-type: none"> <li>• Key characteristics of the graph include any of the following that apply: vertices, major and minor axes and their lengths, foci, directrix, eccentricity, asymptotes, and center.</li> <li>• Determine which conic section is represented by a quadratic equation based on the relative size and sign of the parameters in the equation.</li> </ul>
<p>b. Represent conic sections whose axes are parallel to the x- and y-axes using graphs, verbal statements, and equations. Translate among these representations. Represent the equations of conic sections in multiple forms to extract information about the parabola, circle, ellipse, or hyperbola.</p>	<ul style="list-style-type: none"> <li>• Determine the graph of a conic section whose axes are parallel to the x- and y-axes from its equation, an equation from its graph, and either the equation or graph from verbal information regarding its position and/or characteristics (e.g. vertices, foci, radius, or major and minor axis length.)</li> <li>• Translate from the general form for conic section, <math>Ax^2 + By^2 + Cx + Dy + E = 0</math>, to the standard form for graphic analysis.</li> <li>• Communicate understanding of the relationship between the standard algebraic forms and graphical characteristics for parabolas, circles, ellipses, and hyperbolas.</li> <li>• The standard form of an ellipse <math>\frac{(x-h)^2}{16} + \frac{(y-k)^2}{9} = 1</math> identifies the center as <math>(h, k)</math> and major axis as the x-axis; it provides sufficient information to determine the location of the vertices and foci and the eccentricity.</li> </ul>

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<p>c. Describe the effect that changes in the parameters of a particular conic section have on its shape and position.</p>	<ul style="list-style-type: none"><li>• Parabolas should be limited to those defined by reference to focus, directrix, and/or eccentricity or to non-function parabolas, that is those that can be represented in the form, <math>x = ay^2 + by + c</math>.</li><li>• Parabolas in the form <math>y = ax^2 + bx + c</math> are assessed in the core exam in Polynomial and Rational Functions.</li></ul>
<p>d. Recognize, express, and solve problems that can be modeled using conic sections. Interpret their solutions in terms of the context of the problem.</p>	<ul style="list-style-type: none"><li>• Some real world applications using conic sections may include satellite dish design, planet or satellite orbits, whispering galleries, tracking systems, parabolic mirrors, telescopes, or long range navigational systems.</li><li>• This includes using and interpreting appropriate units of measurement and precision for the given application.</li></ul>

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**Module: Sequences and Series**

This module addresses the patterns in arithmetic and geometric sequences and series. Students are expected to apply the formulas for finding the  $n$ th term of a sequence or series, the  $n$ th partial sum of finite series, and the infinite sum of a geometric series when it exists. General iterative relationships and recursive models are applied to patterns and problems. There will be a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and solve rich contextual problems, where appropriate.

Content Benchmarks	Explanatory Comments and Examples
<b>I 1. Arithmetic and geometric sequences and series</b>	
a. Represent the general term of an arithmetic or geometric sequence and use it to generate the sequence or determine the value of any particular term.	<ul style="list-style-type: none"> <li>• Use and interpret sigma notation, <math>\Sigma</math>, to indicate summation.</li> <li>• Recognize that the sequence defined by: First term = 5; Each term after the first is 6 more than the preceding term as the sequence whose first seven terms are 5, 11, 17, 23, 29, 35, and 41.</li> <li>•</li> </ul>
b. Represent partial sums of an arithmetic or geometric sequence and determine the value of a particular partial sum or sum of a finite sequence.	<ul style="list-style-type: none"> <li>• Use the formulas for sums of finite arithmetic and geometric series to find partial sums or the sum of <math>n</math> terms.</li> </ul>
c. Recognize when an infinite geometric sum can be determined and determine the sum when possible.	<ul style="list-style-type: none"> <li>• Determine that the sum exists when the common ratio, <math>r</math>, is between 1 and -1. (<math> r  &lt; 1</math>)</li> <li>• To find the infinite sum, when it exists, use the formula <math display="block">S_{\infty} = \frac{a}{1 - r}</math>, where <math>a</math> represents the first the first term of the geometric series and <math>r</math> is the common ratio.</li> </ul>
d. Convert the recursive model for linear growth ( $a_1 = a$ , $a_{n+1} = a_n + d$ , where $a$ is the first term and $d$ is the constant difference) to a closed linear form ( $a_n = a + (n - 1)d$ ).	
e. Convert the recursive model of geometric growth ( $p_1 = a$ , $p_{n+1} = rp_n$ where $a$ is the first term and $r$ is the constant growth rate) to a closed exponential form ( $p_n = ar^{n-1}$ ).	
f. Recognize, express, and solve problems that can be	<ul style="list-style-type: none"> <li>• Possible applications are home mortgage problems and other</li> </ul>

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<p>modeled using a finite geometric series. Interpret their solutions in terms of the context of the problem.</p>	<p>compound interest problems.</p> <ul style="list-style-type: none"> <li>This includes using and interpreting appropriate units of measurement and precision for the given application.</li> </ul>
<b>I2. Other types of iteration and recursion</b>	
<p>a. Use recursion to generate and describe, analyze, and interpret patterned relationships other than arithmetic or geometric sequences.</p>	<ul style="list-style-type: none"> <li>Analyze and explain the iterative steps in standard algorithms for arithmetic</li> <li>Use recursion to generate and describe patterned relationships.</li> <li>Analyze the sequences produced by recursive calculations using spreadsheets.</li> </ul> <p><i>Example:</i> The result of repeatedly squaring a number between -1 and 1 appears to approach zero; the result of repeatedly squaring a number less than -1 or greater than 1 appears to continue to increase; determine empirically how many steps are needed to produce 4-digit accuracy in square roots by iterating the operations, divide and average.</p> <ul style="list-style-type: none"> <li>Describe the factorial function or the Fibonacci sequence recursively.</li> </ul>
<p>b. Use iterative methods to solve problems.</p>	<ul style="list-style-type: none"> <li>Display the effect of iteration using a spreadsheet.</li> </ul> <p><i>Example:</i> Use a spreadsheet to display and compare the effect of compound interest on loans or investments at different rates of interest; identify the diminishing effect of increasing the number of times per year that interest is compounded.</p>

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**ADP Algebra II End-of-Course Exam Calculator Policy:** The ADP Algebra II End-of-Course Exam is designed to take 90-120 minutes for most students to complete. The test will be administered in two sessions. The first session will last an estimated 45-60 minutes and will be completed by students without the use of a calculator. The second session will also last an estimated 45-60 minutes and will be completed by students using a calculator (preferably a graphing calculator). Some items in the calculator session assume access to a graphing calculator; therefore graphing calculators are recommended although not required. The following policy provides guidance on the use of calculators during the administration of the second session of the Algebra II End-of-Course Exam. The test will be designed so that some but not all questions on the calculator portion of the test require the use of a calculator.

The following types of calculators are permitted for use:

- graphing calculators (recommended)
- scientific calculators (not recommended)
- four-function calculators (not recommended)

The following types of calculators are not permitted for use on the ADP Algebra II End-of-Course Exam:

- **Texas Instruments: All model numbers that begin with TI-89 or TI-92; Voyage 200; N-Spire CAS**
- **Hewlett-Packard: hp 48GII and all model numbers that begin with hp 40G, hp 49G, or hp 50G**
- **Casio: Algebra fx 2.0, ClassPad 300, and all model numbers that begin with CFX-9970G**
- Calculators with built-in computer algebra systems
- Pocket organizers
- Handheld or laptop computers
- Calculators built into cellular phones or other electronic communication devices
- Calculators that have pen input/stylus-driven devices
- Calculators requiring access to an electrical outlet
- Calculators that make noises of any kind that cannot be noise disabled (except for students needing special accommodations)
- Calculators that use a QWERTY keyboard
- Calculators that use paper tape

Proctors are required to disallow the use of any of the above types of calculators. Substitute calculators may be provided in the event that a student's calculator is disallowed and the proctor has approved calculators available.

Students using a calculator with a raised display or a display one inch or larger will be seated at the discretion of the proctor.

Proctors are required to check calculators before the exam. Students should be thoroughly familiar with the operation of the calculator they plan to use on the exam. Calculators may not be shared, and communication between individual student calculators is prohibited during the exam. Students should use their calculator on a regular basis so that they become adept at using the calculator that they plan to use during the test.